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SUBTRACTION

Mia had 14 baseball cards. She gave 5 away and kept the rest for herself. How many cards did Mia have left for herself?

STEP ONE: Read the entire problem.

“Mia had 14 baseball cards. She gave 5 away and kept the rest for herself. How many cards did Mia have left for herself?”

STEP TWO: Decide who is involved in the problem.

Mia

STEP THREE: Decide what is involved in the problem.

Mia’s
baseball
cards

STEP FOUR: Draw unit bars of equal length.

Mia’s
baseball
cards



STEP FIVE: Read each sentence, one at a time.

A

“Mia had 14 baseball cards.”

Mia’s
baseball
cards



14

“Teacher Talk”

STEP ONE: Let’s read the entire problem.

STEP TWO: Who is involved in this problem? Yes, we’re talking about Mia. Is anyone else involved in this problem? No, only Mia. Let’s write “Mia” on the left side of our work.

STEP THREE: Now that we know who we are talking about, we can ask, “Mia’s what?” What are we talking about? Is it Mia’s lunch? Her stickers? That’s correct: we’re talking about her baseball cards. We need to write “baseball cards” right below Mia’s name. Why did we use an apostrophe and an “s”? That’s correct: to show that the baseball cards belong to Mia.

STEP FOUR: In this next step, we need to draw a unit bar directly to the right of the variable. How many variables do we have? So how many unit bars do we need? Great! We have just 1 variable, so we need just 1 unit bar.

STEP FIVE—A: Let’s read the first sentence. What information does it give us? That’s right: We learn that Mia had 14 baseball cards. Is this her total number of baseball cards? Where should we write the total? Yes, let’s write the total to the right of the unit bar.

B

“She gave 5 away . . .”

Mia’s
baseball
cards



B: Now, let’s go on to the second sentence. What’s the problem with this sentence? Too much information! So let’s “chunk” it, or break it into pieces. If we read just the first part, it says, “She gave 5 away.” Let’s mark off a part of our unit bar, near the end, and let’s write a 5 inside that piece. Should we also give that piece a label, so we can remember what it shows? What would be a good label? “G” for “gave away”? Sounds good to me!

Now let’s think about what’s happening here. What’s Mia doing? Is she going to have more cards or fewer cards? That’s right: fewer. If we want to show that she’s ending up with fewer cards, we’re going to have to cross some out. So let’s put a slash through the part with the 5 to show that those 5 are gone. Does everyone understand what we’re doing? When we’re subtracting, we mark off a segment at the end of the unit bar, and we put a slash through it to show that that part’s gone away; we got rid of it.

C: Now let’s finish the sentence. What does it tell us? She kept the rest for herself. Do we need to change anything in our unit bar to show that? No, we do not. How about adding a label to the other part of the unit bar? What would be a good label to add? “K” for “kept”? Absolutely!

D: We’re ready for the last sentence: “How many cards did Mia have left for herself?” That brings us to the question.

C

“ . . . and kept the rest for herself.”

Mia’s
baseball
cards



D

“How many cards did Mia have left for herself?”

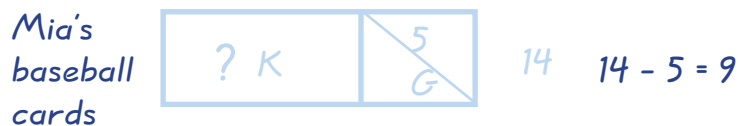
HOLD THIS THOUGHT

- To show subtraction, mark off the appropriate segment of the unit bar and draw a diagonal slash mark through it.

STEP SIX: Put the question mark in place.



STEP SEVEN: Work computation to the side or underneath.



STEP EIGHT: Answer the question in a complete sentence.

Mia had 9 baseball cards left for herself.

STEP SIX: Are we looking for the total number of Mia's baseball cards? No, we already know that she had a total of 14. What are we being asked to do in this problem? You're correct: we're asked to figure out how many baseball cards Mia had left for herself after she gave 5 away.

Who can tell us what we're trying to solve in mathematical language? Great! We're trying to figure out what part of 14 was left after Mia gave 5 cards away. And which part of the unit bar shows what we're trying to figure out? Correct again. Let's put the question mark inside the first part of the unit bar.

STEP SEVEN: Who recognizes what kind of a problem we have? Yes, it's a "whole-part-part." We are given the total and 1 part, so we need to figure out the other part. What should we do to solve this? That's correct: we're going to subtract. How do you know? Now, who can tell me what we'll subtract? What numbers do we have in this problem?

Great! Let's write " $14 - 5 = 9$." How many cards did Mia have left for herself? 9! Correct!

STEP EIGHT: Now we can finish the problem. How should this question be answered in a complete sentence? Very good. "Mia had 9 baseball cards left for herself."

MILKING THE PROBLEM FOR ALL IT'S WORTH

➡ In the end, how many more cards did Mia keep than she gave away?

➡ After she gave 5 cards away, how many cards would Mia need if she wanted 30 in her collection?

RATIO

The ratio of Ty's books to Ling's books is 3 : 4. Ty has 60 books. If Ty buys another 5 books, what will be the new ratio of Ty's books to Ling's books?

.....

STEP ONE: Read the entire problem.

“The ratio of Ty's books to Ling's books is 3 : 4. Ty has 60 books. If Ty buys another 5 books, what will be the new ratio of Ty's books to Ling's books?”

STEP TWO: Decide who is involved in the problem.

Ty

Ling

STEP THREE: Decide what is involved in the problem.

Ty's
books

Ling's
books

STEP FOUR: Draw unit bars of equal length.

Ty's
books 

Ling's
books 

“ Teacher Talk ”

STEP ONE: Let's read the problem.

STEP TWO: Who should we list first? Ty, yes! Why should he be listed first? Who should be listed second? Ling, yes! Is there a third variable? No.

STEP THREE: What are we talking about? We have Ty's and Ling's what? Books!

STEP FOUR: How many bars need to be drawn for this particular problem? How many variables do we have? So we need 2 unit bars, 1 for Ty's books and 1 for Ling's books.

STEP FIVE: Read each sentence, one at a time.

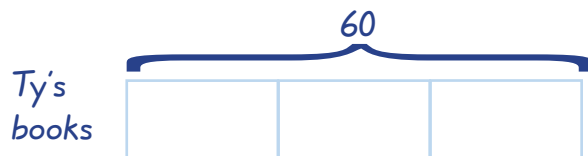
A

“The ratio of Ty’s books to Ling’s books is 3 : 4.”



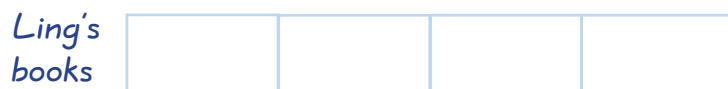
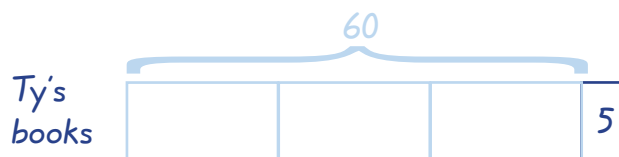
B

“Ty has 60 books.”



C

“If Ty buys another 5 books, . . .”



D

“. . . what will be the new ratio of Ty’s books to Ling’s books?”

STEP FIVE—A: Let’s read the first sentence: “The ratio of Ty’s books to Ling’s books is 3 : 4.” At this time, our unit bars show the ratio being 1 : 1. Does the problem ask for the ratio to be 1 : 1? No. This problem asks for 3 : 4. We need how many more bars added onto Ty’s first bar? Great! The answer is 2. Therefore how many more bars do we need to add to Ling’s first bar? Great! The answer is 3.

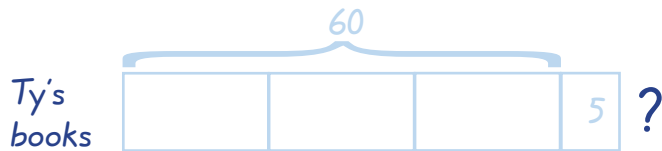
We need to make sure all bars are the same size. In this case, we need to make sure Ty’s unit bars are the same size as what? The same size as Ling’s, so we can compare them.

B: Let’s continue reading: “Ty has 60 books.” Where do we need to put the number 60? Let’s put a brace over Ty’s three units and add a label for “60.” We’re not putting the 60 at the end because earlier we read that he’s going to be getting more. We need to leave room to add more.

C: Go on to the next sentence: “If Ty buys another 5 books, . . .” We pause because of what? The comma! And now we add a bar worth 5 to Ty’s bars. Is 5 as big as 60? No, so the new bar should be much smaller than what we already have.

D: Let’s finish that sentence: “. . . what will be the new ratio of Ty’s books to Ling’s books?” That brings us to Step Six.

STEP SIX: Put the question mark in place.



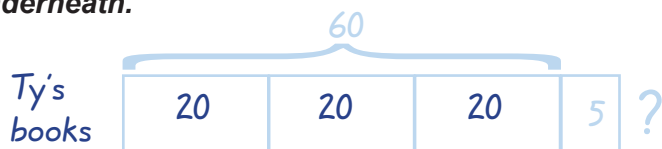
STEP SIX: Since we are dealing with a problem that asks for the ratio of 1 variable to another, we need to put in 2 question marks, 1 for Ty's total and 1 for Ling's total. Now we can compute.

STEP SEVEN: How can we solve this problem? First, we know that the number 60 in our drawing represents how many units? That's right: 3! Therefore we can divide 60 by 3 to get a quotient of 20. So we know that 1 unit equals 20.

How many units in our drawing are the same size? If we know each of Ty's is worth 20, what do we know about Ling's? Great! So we place "20" in each of our 7 equal units. We now notice that Ty has $20 + 20 + 20 + 5$ books and Ling has $20 + 20 + 20 + 20$ books. The new ratio appears to be $65 : 80$.

Can we leave our answer as $65 : 80$? What can we do to give a better answer? Let's simplify! So how do we simplify? That's right! Divide both the 65 and the 80 by the greatest common factor to reach the new answer. What's the greatest common factor? 5? Right! Do we get $13 : 16$ as the new ratio? We sure do!

STEP SEVEN: Work computation to the side or underneath.



$$\begin{aligned} 3 \text{ units} &= 60 \\ 60 \div 3 &= 20 \\ 1 \text{ unit} &= 20 \end{aligned}$$

$$\begin{aligned} 20 + 20 + 20 + 5 &= 65 \\ 20 + 20 + 20 + 20 &= 80 \\ 65 : 80 \end{aligned}$$

$$\begin{aligned} 65 \div 5 &= 13 \\ 80 \div 5 &= 16 \end{aligned}$$

$$13 : 16$$

HOLD THIS THOUGHT

- When dealing with a problem that asks for the ratio of one variable to another, use two question marks.

STEP EIGHT: Answer the question in a complete sentence.

The new ratio of Ty's books to Ling's books will be 13 : 16.

STEP EIGHT: *How would you answer the question in a complete sentence? "The new ratio of Ty's books to Ling's books will be 13 : 16."*

MILKING THE PROBLEM FOR ALL IT'S WORTH

▮▮▮▮▮▶ What is the fraction that would represent Ty's books relative to the total of Ty's books plus Ling's books?

▮▮▮▮▮▶ Suppose that instead of 5 books, Ty buys another 15 books. What would be the new ratio of Ty's books to Ling's books?

FRACTIONS

The candy shop made 400 Valentine candies. The shop sold $\frac{5}{8}$ of them before Valentine's Day and $\frac{1}{5}$ of the remainder on Valentine's Day itself. How many Valentine candies were left after Valentine's Day?

FRACTIONS

Evelyn read 25 pages of a book on Friday morning. She read $\frac{1}{4}$ of the remainder on Friday afternoon. If she still had 90 pages to read after Friday afternoon, how many pages were in the book?

FRACTIONS

The candy shop made 400 Valentine candies. The shop sold $\frac{5}{8}$ of them before Valentine's Day and $\frac{1}{5}$ of the remainder on Valentine's Day itself. How many Valentine candies were left after Valentine's Day?

S	S	S	S	S	50	50	50
50	50	50	50	50	50	50	50

Valentine candies 400

↓

Val.				
Day	30	30	30	30

150

}
?

8 units = 400
 $400 \div 8 = 50$
 1 unit = 50
 $3 \times 50 = 150$

5 units = 150
 $150 \div 5 = 30$
 1 unit = 30
 $30 \times 4 = 120$

After Valentine's Day, 120 Valentine candies were left.

FRACTIONS

Evelyn read 25 pages of a book on Friday morning. She read $\frac{1}{4}$ of the remainder on Friday afternoon. If she still had 90 pages to read after Friday afternoon, how many pages were in the book?

Fri.				
morn.	25	120		

Evelyn's pages read ?

↓

Fri.				
after.	30	L	L	L
	30	30	30	30

120

}
90

3 units = 90
 $90 \div 3 = 30$
 1 unit = 30
 $4 \times 30 = 120$
 $25 + 120 = 145$

There were 145 pages in the book.